DAWES HICKS LECTURE ON PHILOSOPHY

# LEIBNIZ AND DESCARTES: PROOF AND ETERNAL TRUTHS

By IAN HACKING

*Read* 6 June 1973

LEIBNIZ knew what a proof is. Descartes did not. Due attention to this fact helps resolve some elusive problems of interpretation. That is not my chief aim today. I am more interested in prehistory than history. Leibniz's concept of proof is almost the same as ours. It did not exist until about his time. How did it become possible? Descartes, according to Leibniz, furnished most of the technology required for the formation of this concept, yet deliberately shied away from anything like our concept of proof. I contend that Descartes, in his implicit rejection of our idea of proof, and Leibniz, in his excessive attachment to it, are both trying to meet a fundamental malaise in seventeenth-century epistemology. I speak of a malaise rather than a problem or difficulty, for it was not formulated and was perhaps not formulable. But although these unformulated preconditions for the concept of proof are forgotten and even arcane, many facts of the resulting theories of proof are familiar enough. Leibniz was sure that mathematical truth is constituted by proof while Descartes thought that truth conditions have nothing to do with demonstration. We recognize these competing doctrines in much modern philosophy of mathematics. The way in which the two historical figures enacted many of our more recent concerns has not gone unnoticed: Yvon Belaval deliberately begins his important book on Leibniz and Descartes with a long chapter called 'Intuitionisme et formalisme'.[1] There are plenty more parallels there for the drawing. I find this no coincidence, for I am afflicted by a conjecture, both unsubstantiated and unoriginal, that the 'space' of a philosophical problem is largely fixed by the conditions that made it possible. A problem is individuated only by using certain concepts, and the preconditions for the emergence of those concepts are almost embarrassingly determining of what can be done with them. Solutions, countersolutions, and dissolutions are worked out in a space

[1] *Leibniz critique de Descartes*, Paris, 1960.

whose properties are not recognized but whose dimensions are as secure as they are unknown. I realize that there is no good evidence for the existence of conceptual 'space' nor of 'preconditions' for central concepts. Nothing in what follows depends on succumbing to the conjecture that there are such things. The Dawes Hicks lecture is dedicated to history and I shall do history, but I do want to warn that my motive for doing so is the philosophy of mathematics and its prehistory.

In saying that Leibniz knew what a proof is, I mean that he anticipated in some detail the conception of proof that has become dominant in our century. He is commonly said to have founded symbolic logic. He occupies the first forty entries in Alonzo Church's definitive *Bibliography of Symbolic Logic*. I do not have that logical activity in mind. Most seventeenth-century wrestling with quantifiers, relations, combinatorics, and the syllogism seems clumsy or even unintelligible to the most sympathetic modern reader. In contrast Leibniz's ideas about proof sound just right.

A proof, thought Leibniz, is valid in virtue of its form, not its content. It is a sequence of sentences beginning with identities and proceeding by a finite number of steps of logic and rules of definitional substitution to the theorem proved.[1] He experimented with various rules of logic and sometimes changed his mind on which 'first truths' are admissible. He was not able to foresee the structure of the first order predicate logic. He unwittingly made one of our more beautiful theorems—the completeness of predicate logic—into a definition through his equivalence between provability and truth in all possible worlds. My claim for Leibniz is only that he knew what a proof was. He was not even good at writing down proofs that are formally correct, for by nature he was hasty, in contrast to Descartes who despised formalism and who is nearly always formally correct.

The Leibnizian understanding of proof did not much exist before his time. Yet so well did Leibniz understand proof that he could offer metamathematical demonstrations of consistency using the fact that a contradiction cannot be derived in any

[1] This frequently occurring theme is expressed, for example, in the letter to Conring of 19 Mar. 1678, *P.* I, 194. See also *P.* VII, 194 and *O.* 518. On the importance of form rather than content, see the letters to Tshirnhaus, e.g. May 1678, *M.* IV, 451. (*P.* = *Die Philosophischen Schriften von G. W. Leibniz*, ed. G. Gerhardt. *O.* = *Opuscules et fragments inédits de Leibniz*, ed. L. Couturat. *M.* = *Mathematische Schriften*, ed. G. Gerhardt.)

number of steps from premisses of a given form.[1] He understood that a proof of a necessary proposition must be finite, and made an important part of his philosophy hinge on the difference between finite and infinite proofs. We owe to him the importance of the definition of necessity as reduction to contradiction, and the corresponding definition of possibility as freedom from contradiction, understood as the inability to prove a contradiction in finitely many steps. Proof is not only finite but computable, and the checking of proofs is called a kind of arithmetic. Leibniz even saw the importance of representing ideas and propositions by a recursive numbering scheme.[2] His invention of topology is motivated by a theory of the notation needed for valid proof.[3] He is not alone in any of these observations but he did have the gift of synthesizing and stating some of their interconnections. In asking how these ideas became possible it is immaterial whether they are the ideas of a single man. It suffices that they are novel and become widespread in the era of Leibniz, but it is convenient to have an Olympian figure who so perfectly epitomizes this new understanding.

Leibniz himself has a plausible explanation of why the concept of proof emerged at this time. Insight into the nature of proof is not to be expected when geometry is the standard of rigour. Geometrical demonstrations can appear to rely on their content. Their validity may seem to depend on facts about the very shapes under study, and whose actual construction is the aim of the traditional Euclidean theorems. A Cartesian breakthrough changed this. Descartes algebrized geometry. Algebra is specifically a matter of getting rid of some content. Hence in virtue of Descartes' discovery, geometrical proof can be conceived as purely formal. Leibniz thought that Descartes had stopped short, and did not see his way through to a completely general abstract Universal Characteristic in which proofs could be conducted,

> and which renders truth stable, visible and irresistible, so to speak, as on a mechanical basis . . . Algebra, which we rightly hold in such esteem, is only a part of this general device. Yet algebra accomplished this much —that we cannot err even if we wish and that truth can be grasped as if pictured on paper with the aid of a machine. I have come to understand that everything of this kind which algebra proves is due only

[1] For example in notes written in Nov. 1676, intended for discussion with Spinoza. *P.* VII, 261.

[2] *Lingua Generalis*, Feb. 1678, *O.* 277. Cf. L. Couturat, *La Logique de Leibniz*, Paris, 1901, ch. III.

[3] To Huygens, 8 Sept. 1679, *M.* II, 17; cf. *P.* V, 178.

to a higher science, which I now usually call a *combinatorial characteristic*.[1]

'Nothing more effective,' Leibniz ventures to say, 'can well be conceived for the perfection of the human mind.' Insight becomes irrelevant to recognizing the validity of a proof, and truth has become 'mechanical'. Two trains of thought parallel this conception of proof. One has long been known: Leibniz's belief that there exists a proof, possibly infinite, for every truth. Sometimes readers have inferred that the Universal Characteristic was intended to settle every question whereas in fact Leibniz continues the letter quoted above saying that after the Characteristic is complete, 'men will return to the investigation of nature alone, which will never be completed'. The second train of thought concerns probability. Leibniz did often say that when the Characteristic is available disputes would be resolved by calculation. Sometimes these calculations would be *a priori* demonstrations but more usually they would work out the probability of various opinions relative to the available data. In surprisingly many details Leibniz's programme resembles the work of Rudolf Carnap on inductive logic.[2] I shall argue at the end of this lecture that the Leibnizian conceptions of proof and probability have intimately related origins. For the present I shall restrict discussion to proof.

Although the conception of proof and probability is partly familiar, there is a point at which most admirers of Leibniz stop:

> Every true proposition that is not identical or true in itself can be proved *a priori* with the help of axioms or propositions that are true in themselves and with the help of definitions or ideas.[3]

'Every' here includes all contingent truths. Moreover, Leibniz thought one does not fully understand a truth until one knows the *a priori* proofs. Since the 'analysis of concepts' required for proof of contingent propositions is 'not within our power', we cannot fully understand contingent truths. In these passages Leibniz is not giving vent to some sceptic's claim that only what is proven is reliable. Leibniz is no sceptic. He is not even an epistemologist. You need a proof to understand something because a proof actually constitutes the analysis of concepts which in turn determines the truth, 'or I know not what truth

[1] To Oldenburg, 28 Dec. 1675, *M.* I, 84.

[2] For references see my 'The Leibniz-Carnap program for inductive logic', *Jl. Phil.* lxviii, 1971, 597.

[3] *P.* VII, 300.

is'.[1] Moreover a proof gives the reason why something is true, and indeed the cause of the truth. Truth, reason, cause, understanding, analysis, and proof are inextricably connected. It is part of my task to trace the origin of these connections. The connections are not automatic then or now. To illustrate this we need only take the contrasting doctrines of Descartes.

Leibniz thought that truth is constituted by proof. Descartes thought proof irrelevant to truth. This comes out nicely at the metaphorical level. Leibniz's God, in knowing a truth, knows the infinite analysis and thereby knows the proof. That is what true knowledge is. Leibniz's God recognizes proofs. Descartes' God is no prover. A proof might help a person see some truth, but only because people have poor intellectual vision. It used to be held that angels did not need to reason. Although commendably reticent about angels, Descartes has just such an attitude to reasoning. He is at one with the mathematician G. H. Hardy,

> Proofs are what Littlewood and I call gas, rhetorical flourishes designed to affect psychology . . . devices to stimulate the imagination of pupils.[2]

Naturally Descartes says little about demonstration. Much of what he says is consistent with the doctrines advanced in the *Regulae*. Intuition and deduction are distinguished. Elementary truths of arithmetic can be intuited by almost anyone. Consequences may also be intuited. Deduction requires the intuition of initial propositions and consequential steps. The modern reader tends to equate intuition and deduction with axiom and theorem proved, but this is to see matters in a Leibnizian mould. The Cartesian distinction is chiefly psychological. One man might require deduction where another would intuit. In either case the end product is perception of truth. Some Cartesian scholars have recently debated whether the *cogito ergo sum* is inference or intuition or something else again.[3] Descartes does give varying accounts of this famous *ergo* but it is completely immaterial to him whether one man needs to infer where another intuits directly. The point of the *cogito*, as the *Discourse*

[1] To Arnauld, 14 July(?), 1686, *P.* II, 56.

[2] 'Mathematical Proof', *Mind*, xxviii, 1928, 18.

[3] For example, H. G. Frankfurt, 'Descartes' discussion of his existence in the second meditation', *Phil. Rev.* 1966, 333. A. Kenny, *Descartes*, New York, ch. 3. Jaako Hintikka, '*Cogito ergo sum*, inference or performance?' *Phil. Rev.* lxxi, 1962, 3–32. I agree with André Gombay, from whom I have much profited in conversation about Descartes. '*Cogito ergo sum*, inference or argument?' in *Cartesian Studies*, ed. R. J. Butler, Oxford, 1972.

informs us, is to display a truth one cannot doubt. Then one may inquire what, in this truth, liberates us from doubt. The intuition/inference/performative controversy is misguided because Descartes is indifferent to what sort of 'gas' induces clear and distinct perception. However you get there, when you see with clarity and distinctness you note that there is no other standard of truth than the natural light of reason. Leibniz, although granting some sense to 'what is called the natural light of reason',[1] inevitably observed that Descartes 'did not know the genuine source of truths nor the general analysis of concepts'.[2]

The Cartesian independence of truth from proof is illustrated by Descartes' unorthodox view on the eternal truths. These comprise the truths of arithmetic, algebra, and geometry, and usually extend to the laws of astronomy, mechanics, and optics. Contemporary authorities like Suarez taught that eternal truths are independent of the will of God. All the eternal verities are hypothetical. If there are any triangles, their interior angles must sum to two right angles. Since God is free to create or not to create triangles, this hypothetical necessity is no constraint on his power.[3] Descartes, although cautious in expressing opinions at odds with received doctrine, disagreed. The eternal truths depend upon the will of God, and God could have made squares with more or fewer than four sides. As we might express it, the eternal truths are necessary, but they are only contingently necessary.

> Even if God has willed that some truths should be necessary, this does not mean that he willed them necessarily, for it is one thing to will that they be necessary, and quite another to will them necessarily.[4]

I very much like the way that Emile Bréhier[5] uses this theory about eternal truth in order to explain away the Cartesian 'circle' alleged, in the first instance, by Arnauld. The circle goes like this: from the clarity and distinctness of the third meditation it follows that God exists, but clarity and distinctness can be counted on only if there is a good God. Many commentators

[1] To Sophia Charlotte, 1702, *P.* VI, 501.

[2] To Philip, Dec. 1679, *P.* IV. 282.

[3] F. Suarez, *Disputationes Metaphysicae*, 1597. Cf. T. J. Cronin, *Objective Being in Descartes and in Suarez*, Analecta Gregoriana 154, Rome, 1966.

[4] To Mesland, 2 May 1644. Other texts on eternal truths are as follows. To Mersenne, 6 May and 27 May 1630 and 27 May 1638. Reply to *Objections* V and VI. *Principles* xlviii–xlix.

[5] 'La creation des verités éternelles', *Rev. Phil.* cxxiii, 1937, 15.

interrupt this simple-minded circle by saying that God's veracity is not needed when we are actually perceiving truth with clarity and distinctness. God comes in only when we turn our minds to another thought. This leaves open the question of the role that God plays when we are thus distracted. There are several competing interpretations. André Gombay uses this comparison.[1] In moments of passionate love a man (such as the husband in Strindberg's play, *The Father*) cannot doubt that his wife is faithful. But at more humdrum moments he doubts her love. What is his doubt? (*a*) His memory is playing tricks; the feeling of passionate certainty never occurred. (*b*) He remembers correctly his passionate conviction, but subsequently feels that he was misled by his passion. No matter how convinced he was then, he was wrongly convinced. (*c*) She was true to him at that passionate moment, but is no longer so. In the case of Cartesian doubt, recent commentators correctly rule out doubts of kind (*a*): God is no guarantor of memory. Gombay, probably rightly, favours (*b*). But doubt of kind (*c*) is instructive. Bréhier proposes that God is needed to ensure that an 'eternal truth', once perceived clearly and distinctly, *stays* true.

No set of texts tells conclusively for or against the Bréhier reading. This in itself shows how far Descartes separates proof from truth. What would happen to the proof of *p* if *p*, previously proven, went false? We can imagine that in the evolution of the cosmos Euclid's fifth postulate was true, relative to some assigned metric, and subsequently ceased to be true. At least this remains, we think: if a complete set of Euclidean axioms is true, the Pythagorean theorem is true too. That necessary connection between axiom and theorem cannot itself be contingent. Descartes disagreed. God is at liberty to create a Euclidean non-Pythagorean universe. We owe to Leibniz the clear statement that if not-*p* entails a contradiction then *p* is necessary and indeed necessarily necessary. Descartes grants that it is unintelligible how *p* can entail contradiction and still be true. But this unintelligibility shows the weakness of our minds. Leibniz caustically dismisses this view of modality.[2] It betrays, he thought, a lack of comprehension of the very concepts of necessity, contradiction, and proof.

Not only did Descartes acknowledge no dependence of necessary truth on proof; he also challenged accepted modes of presenting proof. He favoured 'analysis' rather than 'synthesis'. His

[1] 'Counter privacy and the evil genius', read to the Moral Sciences Club, 30 May 1973.

[2] *Monadology*, § 46.

doctrine is sufficiently hard to understand that Gerd Buchdahl distinguishes radically different Cartesian meanings for 'analysis',[1] but even if Descartes ought to have distinguished meanings of the word, he intended to be unequivocal. Synthesis is deduction, whose paradigm is Euclid. Deduction may bully a reader into agreement, but it does not teach how the theorem was discovered. Only analysis can do that. Descartes subscribed to the standard myth that the Greeks had a secret art of discovery.[2] The new algebraic geometry rediscovered it. He called it analytic geometry, as we still do. Its method is to:

> suppose the solution already effected, and give names to all the lines that seem needful for the construction . . . then, making no distinction between known and unknown lines, we must unravel the difficulty in any way that shows most naturally the relations between these lines, until we find it possible to express a quantity in two ways.[3]

Then we solve the equation. Analysis is a mode of discovery of unknowns, and the arguments of the *Geometry* show how solutions can be obtained. Descartes thought that the physicist postulating causes on the basis of observed effects may be doing analysis, and he maintained that the *Meditations* furnish another example of analysis.

The Cartesian notion of analysis underwent strange transformations. The fact that Euclidean synthesis was deemed to depend on content as well as form is well illustrated by Descartes' own observations that in geometry the primary notions of synthetic proofs 'harmonize with our senses'. The point of all those 'minute subdivisions of propositions' is not even to ensure that the proof is sound. It is to render citation easy 'and thus make people recollect earlier stages of the argument even against their will'.[4] Synthetic proofs work partly because we have sensible representations of what we are proving and are thus unfit for metaphysics which uses abstract concepts. Yet by a strange inversion, it is Cartesian analysis that enables Leibniz to argue that proof is entirely a matter of form, and to apply this thought to deductive proof in general, including synthesis. Moreover, what he calls the analysis of concepts proceeds by what Descartes would have called synthetic demonstration!

Descartes wanted good ways to find out the truth and was indifferent to the logical status of his methods. This is well

[1] *Metaphysics and the Philosophy of Science*, Oxford, 1969, ch. 3.
[2] At the end of the reply to the second set of *Objections*.
[3] From the beginning of the *Geometry*. [4] Op. cit., n. 2.

illustrated by yet another kind of 'analysis'. Traditionally science was supposed to proceed by demonstration of effects from causes stated in first principles. In practice the more successful scientists were increasingly guessing at causes on the basis of effects according to what we can now call 'the hypothetico-deductive method'. When challenged Descartes said that this too is a kind of 'demonstration', at least according to 'common usage', as opposed to the 'special meaning that philosophers give' to the word 'demonstration'. In reality, says Descartes, there are two kinds of demonstration, one from causes to effects, in which we prove the effect from the cause, and the other from effect to cause, in which we explain the effect by postulating a cause.[1]

There was a pressing practical problem for the second kind of so-called demonstration. As his correspondent put it, 'nothing is easier than to fit a cause to an effect'. To which Descartes replied that 'there are many effects to which it is easy to fit separate causes, but it is not always so easy to fit a single cause to many effects'. This thought was worked up by Leibniz into the theory of 'architectonic' reasoning.[2] We seek those hypotheses that would be attractive to the Architect of the World, who has a mania for maximizing the variety of phenomena governed by laws of nature, while minimizing the complexity of those selfsame laws.

On such questions of method there does not seem, in perspective, very much at issue between the two philosophers. But they have radically different theories of what they are finding out. Leibniz supposes that truths are constituted by proof, and so proof is essentially linked to truth, while Descartes imagines that truths exist independently of any proof. However, we shall not find the origin of this difference in what might be called the philosophy of mathematics, but in what we should now call the philosophy of science. The very success of scientific activity in the early seventeenth century had created a crisis in man's understanding of what he knows. In the medieval formulations, adapted from Aristotle, knowledge or science was arrived at by demonstration from first principles. It demonstrated effects from causes, and its propositions were universal in form and were necessarily true. In giving the causes, it gave the reasons for belief, and also the reasons why the proposition proved is true. As well as arithmetic and geometry, science included

[1] To Morin, 13 July 1638.

[2] *Tentamentum Anagogicum*, 1696, *P.* VII, 270.

astronomy, mechanics, and optics. This did not mean that one was supposed to do all one's mechanics *a priori*, for it might need ample experience to grasp the first principles of the universe. Francis Bacon furnishes a good example of a thinker trying to preserve this old ontology, insisting that instead of being dogmatic, the scientist must survey large quantities of experiences before he ventures to guess at the axioms, common notions, and first principles. What one is aiming at, however, is a body of universal and necessary axioms which will, when recognized and understood, have the character of self-evidence.

Bacon's methodology is a despairing attempt to save the old theory of truth on its own ground. Increasingly men of science are not doing what they are supposed to be doing. Among what I shall call the high sciences, astronomy, mechanics, and optics, there is a dogmatic school maintaining the Aristotelian physics. It is shattered by new theories which do not merely contradict the old physics but do not even have the same kind of propositions that the old physics sought after. Moreover, among the low sciences, medicine and alchemy, whose practitioners are what Bacon scornfully called the empirics, there has developed a set of practices and concepts that are unintelligible on the old model of knowledge.

Descartes' curious assertions about 'false hypotheses' illustrate how far he has come from traditional views. He says at length in his *Principles*, and throughout his life to various correspondents, that the chief hypotheses of his physics are strictly false, and may be regarded as a kind of fable.[1] It is common to construe this as a safety net spread out after the Galilean scandal. Is it? Hypotheses serve as the basis for deducing true effects, but are not themselves to be asserted as true. Many ancient writers, including Archimedes, base their demonstrations on hypotheses that are strictly false or so Descartes says. Perhaps he is merely seeking bedfellows in support of political caution. I see no reason to think so. Leibniz says that if they worked Descartes' 'false hypotheses' would be like cryptograms for solving the regularity of phenomena,[2] and he also says that Descartes is just wrong in changing the direction of physics to a search for false hypotheses. In short the Cartesian view was taken literally by the next generation of readers.

If Descartes means what he says everything has been turned upside down. Science was to make the world and its truths

[1] *Principles*, xliii–xlvii, and, e.g., To Mesland, May 1645.

[2] To Conring, 19 Mar. 1678, *P.* I, 194.

intelligible. From universal first principles concerning essence and cause and the true being of things one was to deduce the effects and their reasons, making intelligible the variety of general phenomena present to us. The first principles were to get at the very core of truth. But now the core evaporates, turns into a mere sham, a cryptogram of falsehoods. New merits have to be found for science, chief among them, in the seventeenth century, being the virtue of predictive power. In the traditional theory of truth, predictive power did not matter much because science was demonstrating necessities. When it abandons its ability to give reasons and causes by way of first principles, all it can do is provide us with predictions.

The evaporation of truth is what I have called the malaise or even the crisis in the early seventeenth century. We have been accustomed, especially in Britain, to notice the epistemological worries of the period. In fact men wrote treatises not of epistemology but of methodology. The methodology was an attempt to tell how to do what was in fact being done, and how to do it better. The Cartesian titles such as *Rules for direction of the mind*, or *Discourse on Method*, are characteristic of the time. Underneath these works runs not the problem of British empiricism-scepticism, 'How can I ever know?' It is rather, 'What is knowledge, what is truth, are there such things?'

Reconsider the situation of Descartes. We have usually read him as an ego, trapped in the world of ideas, trying to find out what corresponds to his ideas, and pondering questions of the form, 'How can I ever know?' Underneath his work lies a much deeper worry. Is there any truth at all, even in the domain of ideas? The eternal truths, he tells us, are 'perceptions . . . that have no existence outside of our thought'.[1] But in our thought they are, in a sense, isolated perceptions. They may be systematized by synthesis but this has nothing to do with their truth. The body of eternal truths which encompassed mathematics, neo-Aristotelian physics and perhaps all reality was a closely knit self-authenticating system of truth, linked by demonstration. For Descartes there are only perceptions which are ontologically unrelated to anything and moreover are not even candidates for having some truth outside my mind. One is led, I think, to a new kind of worry. I cannot doubt an eternal truth when I am contemplating it clearly and distinctly. But when I cease to contemplate, it is a question whether there is truth *or* falsehood in what I remember having perceived. Bréhier

[1] *Principles*, I. xlviii.

suggested that demonstrated propositions may go false. It seems to me that Cartesian propositions, rendered lone and isolated, are in an even worse state. Perhaps neither they nor their negations have any truth at all. They exist in the mind only as perceptions. Do they have any status at all when not perceived? When demonstration cannot unify and give 'substance' to these truths, the constancy of a veracious God who wills this truth suddenly assumes immense importance. We have long been familiar with the role of God as the willing agent that causes Berkeley's perceptions. We know Leibniz required the mind of God as the arena in which the essences of possible worlds compete for existence, saying indeed that

> neither the essences nor the so-called eternal truths about them are fictitious but exist in a certain region of ideas, if I may so call it, namely in God himself.[1]

I am suggesting that Descartes' veracious God is needed not just to guarantee our beliefs, but also to ensure that there is some truth to believe. I do not claim this as a worked out Cartesian thought but rather as an underlying response to the breakdown in the traditional conception of knowledge.

Descartes was almost ingenuously radical. Faced by the fact that the new science was not Aristotelian knowledge or *scientia*, he abolished the traditional concepts even where they did work, namely in arithmetic and geometry. Leibniz, in contrast, was ingeniously conservative. The merit of the old system was that it gave us some understanding of the nature and interconnection of truths. The demerit was the inadequacy of the implied methodology of doing physics by deduction. So Leibniz grafted a new methodology on to the old theory of demonstration. Demonstration was formerly the key to both ontology and method. Leibniz restricts it to the former. It is turned into the theory of formal proof. In the old tradition only universal propositions are subject to demonstration. In the new practice, only what we now call pure mathematics fits this model. But Leibniz, making proof a matter of ontology, not methodology, asserts that all true propositions have an *a priori* proof, although in general human beings cannot make those proofs. This is to resolve the open question as to the nature of truth. Hence his careful distinction between finite and infinite proofs, the importance of form over content, and all the rest of Leibniz's rendering truth 'mechanical'. The universal characteristic, you will recall, 'renders truth

[1] 'On the radical origination of things', 23 Nov. 1697, *P.* VII, 305.

stable, visible, and irresistible, as on a mechanical basis'. The new science that was not *scientia* had made truth totally unstable. The concept of formal proof was intended to restore the balance.

The ingenuity of Leibniz's eclecticism shows itself in another direction. The Universal Characteristic, as I have said, was to be the vehicle of finite deductions and of probability calculations of inductive logic. Whereas demonstration is the tool of what was traditionally called knowledge, probability, in medieval times, pertained to a quite different realm, opinion. The low sciences of alchemy and medicine are the artisans of opinion and the forgers of probability—or so I argue at length in a forthcoming book, *The Emergence of Probability*. Those thoroughly alien hermetical figures of the Renaissance did more: they actually engendered a concept of inconclusive evidence derived from facts, as opposed to testimony. The high sciences related to experience in a hypothetico-deductive or one might say 'Popperian' way. That is, they concerned themselves with the deductive connections between experienced effects and conjectured causes. The low sciences were too inchoate for that, and created what, in recent times, has been called probability and induction. Leibniz puts the antique theory of demonstration into the realm of ontology. Finite demonstrations become the topic of mathematics, now rendered formal. Architectonic reasoning is his version of the hypothetico-deductive method. Inductive logic is the rationalization of what Bacon dismissed as mere empiricism. The vehicle for all these parts of methodology is the Universal Characteristic. It is a vehicle that cheerfully carries finite proofs and calculations of probability, and yet is a coarse and inadequate mirror of the very nature of truth, the infinite proof.

Carnap and Popper have recently re-enacted the tension between Leibniz's inductive logic and his architectonic reasoning. My topic today is proof, not probability. I claim that the concept of formal proof was created in the time of Leibniz to overcome quite specific breakdowns in traditional ontology. The Cartesian concept of anti-proof has the same origin. These concepts were devised, almost unwittingly, to fill a vacuum. We still employ those concepts but live in a vacuum that those concepts cannot fill. Consider the sterility of modern philosophy of mathematics—not the collection of mathematical disciplines now called the foundations of mathematics, but our conflicting theories of mathematical truth, mathematical knowledge, and mathematical objects. The most striking single feature of work on this subject in this century is that it is very largely banal. This is despite the ample fertilization from the great programmes and discoveries in the foundations of mathematics. The standard textbook presentations of 'Platonism', constructivism, logicism, finitism, and the like re-enact conceptual moves which were determined by an ancient and alien problem situation, the disintegration of the concept of *scientia* and the invention of the concept of evidence culminating in the new philosophy of the seventeenth century. We have forgotten those events, but they are responsible for the concepts in which we perform our pantomime philosophy.

Take, for example, the most seemingly novel, and also the most passionately disparate of contributions, Wittgenstein's *Remarks on the Foundations of Mathematics*. He invites us to destroy our very speech, and abandon talk of mathematical truth and knowledge of mathematics and its objects. We are asked to try out language in which mathematics is not 'true', our discoveries are not 'knowledge' and the 'objects' are not objects. Despite this fantastic and perplexing attempt to get rid of all these inherited notions, Wittgenstein ends up with a dilemma that is essentially Leibniz-Cartesian. On the one hand he suggests, in quite the most radical way, that mathematical 'truth' is constituted by proof, and on the other he is obsessed by just the intuitions that so impressed Descartes. Hardly anyone thinks he has achieved a synthesis of these notions. There is a reason for this. He rejects that antique tryptich, truth, knowledge, and objects, but works in the space created by that earlier period, and is driven to employ the concepts created then for the solution of quite other problems, and which are fettered by their need to solve those other problems. The 'flybottle' was shaped by prehistory, and only archaeology can display its shape.