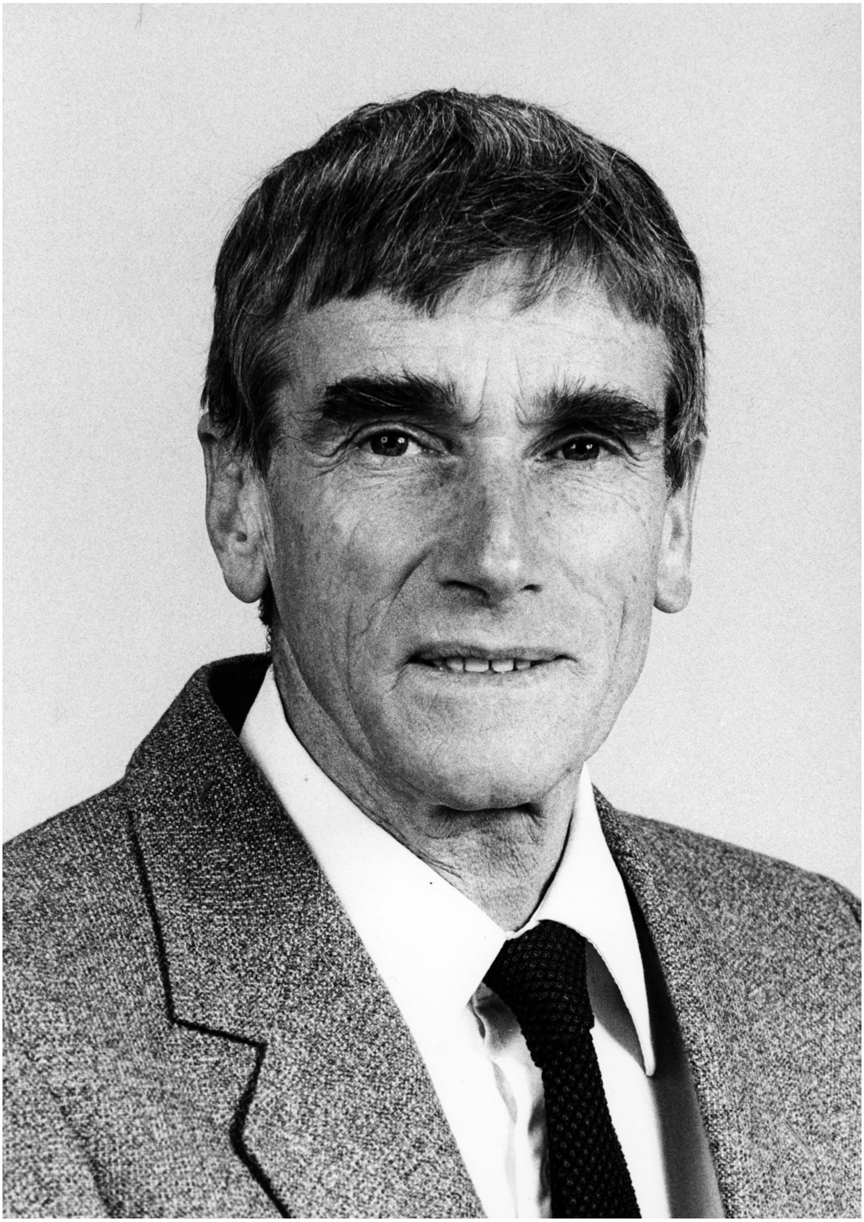

JOHN CEDRIC SHEPHERDSON

John Cedric Shepherdson
1926–2015

Modest, meticulous, masterful. The first word seems to be mentioned by everyone who knew him. The second is obvious to anyone who reads his work and especially those who worked with him. The last is not adequately remarked on: his range was exceptional. His career comprised not so much in asking (and answering) groundbreaking questions, but more in answering questions others might (or should) have asked but did not; or even questioning what had been done and clarifying—and sometimes correcting—in a way that gave the mathematics extra insights, solidity and depth. He was the master craftsman whose deft touch sometimes perfected, sometimes mended, sometimes completed the work of others and thereby opened the way to the next stage. Yet his modesty meant that each of his achievements was initially known only to a small group, be it colleagues or friends or family. It also means that this memoir's main aim is to continue the process of revelation that began at his memorial service in February 2015, where both family and colleagues were surprised to learn many things they had never known about him.

John Cedric Shepherdson came from a long line of Yorkshire folk on both sides. The male line started out and, for a long time, did not move far from High Catton, about 9 miles east of York, while the female line came from Bradford. John's great-great-grandfather, John Shepherdson, who was born in 1790 or 1791, was a carpenter, but the male line, from his only child, Henry Johnson Shepherdson (1814–95), on, were all teachers with one exception. Henry was a general teacher at the Wesley School and his first wife, whom he married in 1838, was a schoolmistress. She died in

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1861 and he then married Isabella Currey (1833/4–1911) who seems to have been solely engaged in household duties.

Their only son, Burton Shepherdson (1867–1938), was a schoolteacher, as was Burton's wife, Carrie (Caroline Elizabeth Ann Lord, 1867–1936), though she apparently only taught for one year, giving up after marrying. Burton taught wood- and metalwork at Bradford Bellevue High School, the grammar school for boys: 'His hobbies were cabinet making, bronze and copper ware with hand beaten patterns, watercolour painting,—mostly landscapes, and gardening.' After retirement, he started learning French. When he died he left £3,000, a tidy sum for the time.

Their elder child, Arnold Shepherdson (1893–1946), was a notable exception in that he did not teach, but he had a university education, supported by his father and a Queen Victoria Scholarship, reading Chemistry at the University of Leeds. He went on to become a research chemist at Imperial Chemical Industries (ICI).

John Cedric Shepherdson was the third child of Arnold and his wife Elsie Aspinall (1895–1977). Elsie went to a private school, but her father died when she was fourteen and she had to leave school to help her mother support the family. Her male ancestors seem to have been salesmen for a few generations, while the females were occupied with domestic duties. A year after Elsie married Arnold, the couple moved to Manchester in 1920—the first time the Shepherdsons had lived outside Yorkshire. However, their first child, Olive May (1919–91), married Douglas Oswald Neale Raven in 1940 in Manchester and then they emigrated to Canada. Elsie developed Alzheimer's at the age of 74 and moved back to Shipley, Yorkshire.

John was born in Huddersfield on 7 June 1926, but the parents—who were comfortably but not well off—together with their four children soon moved to Manchester, where they lived in a middle-class semi-detached house at 19 Dawlish Road in Chorlton-cum-Hardy. His elder sister, Margaret (1924–61), was also a teacher, in primary school. John's father died in 1946 at the early age of 53 from coronary thrombosis. John's genetic inheritance would catch up with him later.

John's younger sister, Hilda (1930–53), married in 1951 but, after giving birth, suffered from severe post-natal depression. She fell into a canal and drowned. At the inquest the coroner's verdict was 'Misadventure', though this was doubted. Her death had a grave impact on the whole Shepherdson family, even Olive, who was already in Canada by then.

Wet always, grimy then though not now, was the Manchester where John Shepherdson grew up. Following the end of the war there was a depressing time when food rationing and other restrictions continued. Reports indicate that he did not have a particularly happy childhood. He failed to gain a scholarship in the then eleven-plus examination to Manchester Grammar School when he was ten, but the following year he succeeded and he remained there from 1937 to 1943. Manchester Grammar School has long been known as a forcing ground for bright young students and Mr Heywood, then head of mathematics, worked his pupils hard. They covered a vast range of mathematics, both pure and applied, and including much geometry, a subject that subsequently went out of fashion. Indeed, this was not just any old geometry but, as John put it, ‘that hardest of all geometries, that good old nineteenth-century subject, geometrical conics, where you prove complicated properties of conics from the focus directrix definition’. The hard work paid off, with fifteen out of twenty-three boys getting open scholarships or exhibitions to Oxford or Cambridge. John was top in mathematics at Manchester Grammar, top in the Northern Universities Joint Matriculation Board Higher School Certificate and awarded a State Scholarship as well as winning an Open Scholarship—the top one in fact—to Trinity College, Cambridge. However, John’s mathematical abilities had not been forced; it became clear they were inherent.

At the age of seventeen he went up to Cambridge, which would bring dramatic changes to his life after Manchester. It was still wartime and he read mathematics for two years, completing Part Two of the three-part Mathematical Tripos, in which he was told unofficially by Hans Heilbronn, then a Fellow at Trinity, that he was top. The title of Senior Wrangler had already been abolished; otherwise he would have had that. Although he could only attend Part Three lectures for the first term of the academic year 1945–6 he nevertheless got First Class Honours and his degree was awarded in 1946. The range of mathematics that he studied for Part Three was already indicative of his remarkable mathematical virtuosity. Most mathematicians are content to work in one broad area—say algebra or analysis or fluid mechanics. John went to Goldstein’s lectures on aerofoils and propeller theory and Dirac’s lectures on quantum theory, but he also dabbled in number theory. He admitted that this was not ‘up his street’; nevertheless he would tackle problems in that area, not always with success, as will be seen.

The first lecture on logic he heard was from Ludwig Wittgenstein. This took place in Wittgenstein’s rooms in Trinity, in the same court that John had rooms. As others have recounted, the room was quite bare of furniture except for a few deckchairs. Twenty or thirty people were present, which

Wittgenstein thought was too many. 'Why do you come here?' he said, 'You won't understand.' Shepherdson was convinced of this by the end of the first lecture and did not return; Georg Kreisel, who was later to be a significant influence on Shepherdson and many other logicians, completed the course, as did John's future colleague at Bristol, Stephan Körner. Despite this introduction to logic he went to S. W. P. Steen's course on mathematical logic, though Steen's main interest was in a completely different area of mathematics, namely complex function theory. He used Quine's book *Mathematical Logic*, which John found very boring. Nevertheless John bought a copy of the book halfway through the course and found in it an erratum slip that said the logical system employed was inconsistent. He pointed this out to Steen, but it is not clear this had any effect. Other logicians would have been dismayed.

John later noted that this, his first real exposure to detailed mathematical logic, mainly comprised proving boring formal theorems in an inconsistent system. So far logic had no great attraction. After only one term of his third year in Cambridge he was 'directed by the Joint Recruiting Board to the National Physical Laboratory [NPL at Teddington] to work for the Ministry of Aircraft Production'. There he undertook his first mathematical research—in fluid mechanics.

The years at Cambridge transformed John Shepherdson's life. It was there that he became involved in various demanding physical pursuits and first developed his lifelong love of the outdoors. He started rock climbing, learnt how to ski, rode a motorbike and had a soft-top sports car. He was also fifth board on the University chess team. He went from having a rather dull life in Manchester to suddenly having incredible freedom to do all these things. Climbing was a great love throughout his life, but he claimed skiing was his greatest sporting pleasure and he went skiing every year even if that meant cutting back on other expenses to fund it. He last went skiing with his son, David, only four years before his death. He also enjoyed cycling, sailing and scuba diving, while indoors he played squash, not to mention chess. His love of the outdoors was not unrelated to his academic activities.

In 1944 John was walking with a Cambridge friend on Crib Goch (Welsh for Red Ridge), which is at one end of the Snowdon Horseshoe, when they encountered Hermann Bondi, who was normally a fellow at Trinity.[1] The young men shared their lunch with him since Bondi looked

[1] John Shepherdson's recollection differs a little from that of Bondi as recorded in the latter's autobiography, *Science, Churchill and Me* (Oxford, 1990).

ill-equipped for mountain walking, but they were amazed when, at the summit of Snowdon, where there was an apparently deserted hotel, Bondi told them that this was where he lived, and would they like to come in for a cup of tea. The story was that Fred Hoyle, a Fellow of St John's, Cambridge, and Bondi had persuaded the Admiralty that this highest point in England and Wales was ideal for studying airborne radar. When Bondi returned to Cambridge after the war he took Shepherdson 'under his wing'. He gave John one of his papers, but John found his knowledge of physics was inadequate and he doubted he would ever become a good enough analyst. This misgiving was reinforced by Besicovitch, who was teaching him analysis. Like many, perhaps most, mathematics students he made the usual naïve mistakes that seem only to be overcome after two or three periods of protracted study of the same bit of theory.

Bondi introduced John to Fred Hoyle and Tommy Gold, both physicists, and John Kendrew, a biochemist, all of renown. However, it was Gold's physical abilities that seem to have impressed John most. In climbing Gold could pull himself up using just one finger from each hand; John never succeeded in using fewer than two, though even in his later years he would compete with his son David in attempting the greater number of one-finger push-ups, and that after dinner. John also prided himself on being able to climb up onto a mantelpiece. He claims that the narrowest he could get onto was only 6.2in. (15.7cm.) wide. Another pursuit of his was to traverse around rooms clinging to the panelling.

But to return to John's developing mathematical career. After three months at the NPL the war had ended but John was still subject to the Joint Recruiting Board, so he could not go back to finish Part Three of his degree. Instead he moved to the Mathematics Department of the NPL. There he worked on numerical analysis, but this was before there were computers, though Alan Turing was working at the NPL building the ACE, the first British computer.

Encouraged by Bondi, who argued that there would be a great need for university lecturers for servicemen returning to tertiary study, John applied for various university positions. He was only nineteen. He tried several places and then accepted a job at Dalhousie in Canada, having turned down an offer from ICI where his father had worked, even though they were prepared to pay for him to return to his studies at Cambridge. But then he was offered a job at Bristol University and deftly cancelled his Dalhousie acceptance.

While at Cambridge John had made up a four at bridge in Trinity with Bondi, Heilbronn, who was subsequently appointed as a Reader at Bristol, and Maurice Pryce, whose son, John, subsequently held a chair in Physics at Bristol. Perhaps it was partly through this acquaintance that Shepherdson came to be appointed to an Assistant Lectureship at Bristol in 1946 where Heilbronn had become head of mathematics. Heilbronn, who was a 'forceful character', to use John's words, continued to be his mentor but other influences were to change the course of his interests: a pattern that would recur throughout his life.

John was younger than many of his student audience since it contained people who had returned from the war and, unsurprisingly, he was often mistaken for a student. He had nine hours of lectures a week, much higher than nowadays, but the classes were small and the marking light and, in those days, the reporting requirements small or almost non-existent. Besides, there were no slides or handouts to prepare, just blackboard work. There was time for research, but John dithered. He had done fluid mechanics at the NPL and he thought he wanted to be a mathematical physicist, but he still felt his knowledge of physics was inadequate.

Heilbronn suggested a problem in so-called elementary number theory. There is nothing elementary in the methods of the subject! It was proposed that Shepherdson start by reading a classical work, Landau's *Vorlesungen über Zahlentheorie*, but John thought there were five volumes of that.[2] Nevertheless he started looking at well-ordered series and this resulted in his second and very substantial paper: 'Well-ordered sub-series of general series'.[3] In 1947 he had already published his first (four-page) paper, 'On the addition of elements of a sequence'.[4] For a fortnight in August that year he went climbing in the Alps for the first time; he went to Arolla in a company having 'many Alpine novices'.

It was his fellow assistant lecturer at Bristol, Gwynne Mostyn, who introduced him to Gödel's work on the Continuum Hypothesis and the Axiom of Choice.[5] Gödel's slim volume was published in the prestigious Princeton Annals of Mathematics series, a series of red paper-covered books that simply reproduced typescripts. Nevertheless these were tremendously influential books. The topics had intrigued mathematicians

[2] In his retirement speech he correctly said there were three volumes.

[3] J. C. Shepherdson, 'Well-ordered sub-series of general series', *Proceedings of the London Mathematical Society, Series* 3, 1 (1951), 291–307.

[4] J. C. Shepherdson, 'On the addition of elements of a sequence', *Journal of the London Mathematical Society*, 22 (1947), 85–8.

[5] K. Gödel, *The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis with the Axioms of Set Theory* (Princeton, NJ, 1940).

since the time of Georg Cantor in the second half of the nineteenth century, who had developed a theory of infinite cardinal numbers, and John's work was a vital stepping-stone to the breakthrough by Paul Cohen in 1963, which subsequently spawned what one might call an industry.

The problem of the Continuum Hypothesis can be adumbrated as follows. When you take a collection of, say, three objects, then you can form eight sub-collections: for any sub-collection, each of the three objects either belongs to it or not. So there are two choices for each and a total of $2 \times 2 \times 2$, or 2^3, = 8 possibilities. This can be extended even to infinite sets—for example, the set of all so-called natural numbers 1, 2, 3 and so on. Unsurprisingly the number of sub-collections is larger than the number of objects. How much larger? The Continuum Hypothesis says it is the next (infinite cardinal) number after the (cardinal) number of the natural numbers. Although this cannot (as we now know) be proved from the axioms of set theory, Gödel showed it was consistent with them.

Abraham Fraenkel, Paul Bernays and Andrzej Mostowski had developed a technique earlier and Gödel built on their work.[6] The technique was to assume the world of (abstract) sets was consistent, then 'define' a smaller collection of sets; in Gödel's case he defined the so-called 'constructible sets'. Next he showed that in this, what we might call mini-universe, the axioms of set theory were still consistent, but in addition the Continuum Hypothesis was true. Using the meticulous approach that he always employed, John Shepherdson took Gödel's monograph and explored the limits of its methodology. He showed that every inner model defined using only constructible sets must also be constructible. This exhibited the limitations of Gödel's approach since in any such 'inner model' the Axiom of Choice and the Continuum Hypothesis still had to be true. Consequently these statements could not be independent of the other axioms of set theory in such a model: their fates were already sealed and some other technique was needed. A very different method had to be introduced, and this was not achieved until more than a decade later when Paul Cohen invented 'forcing' in 1963. John's work resulted in a paper that was published, in three parts, in the *Journal for Symbolic Logic*.[7] His oeuvre is memorable for this alone.

[6] In his research John Shepherdson used what is generally known as Mostowski's collapsing lemma, but he discovered this very elegant result independently of Mostowski.

[7] J. C. Shepherdson, 'Inner models for set theory. I', *Journal of Symbolic Logic*, 16 (1951), 161–90; J. C. Shepherdson, 'Inner models for set theory. II', *Journal of Symbolic Logic*, 17 (1952), 225–37; and J. C. Shepherdson, 'Inner models for set theory. III', *Journal of Symbolic Logic*, 18 (1953), 145–67.

As an assistant lecturer John did not have tenure, so he started looking around for other possibilities. From Bristol he tried to return to Cambridge. He seems to have made three attempts to obtain a Fellowship at Trinity, but lost out to the renowned Peter Swinnerton-Dyer. The famous number theorist, J. E. Littlewood, who worked with G. H. Hardy—also at Trinity—posed a problem. Perhaps foolishly, John attempted it and so did Swinnerton-Dyer, but the latter was an analytic number theorist and solved it, whereas John only made a 'half-hearted attempt'. He stayed at Bristol and was quickly promoted to a lectureship in 1949.

In 1953 John met his future wife, Margaret Smith, who was his junior by four years. After attending schools in Welwyn Garden City, Dartmouth and Newcastle-under-Lyme, she studied biochemistry at St Andrews, obtaining First Class Honours for her BA. She then moved to Bristol to do a PhD and it was in John's lectures on statistics that they met. After their separation owing to Margaret's year in Munich at the Max Planck Institut für Chemie in 1955—for which she had gained a scholarship—and again when John was at Princeton, they were only to be apart briefly once they were married in July 1955.

Gödel, who was at the Institute for Advanced Study at Princeton, may have been influential in arranging John's year there in 1953–4. Gödel was to write much later, on 29 January 1964: 'I know Professor Shepherdson as a very competent mathematical logician.' However, John only saw Gödel once, and that for just an hour, after he had written him a letter saying that he wished to talk about inner models.

At the Institute he shared a room with Martin Davis, who wrote the first book devoted to computability.[8] This was the point at which John started to turn his attention to matters computable or, as they were generally known at the time, recursive functions and procedures. Davis was working on Diophantine equations. These are equations involving powers of variables but the required solution has to be in whole numbers (not all zero). For example, $x^2 - 3x + 2 = 0$ is a Diophantine equation that has solutions $x = 1$ and $x = 2$. On the other hand, Fermat's famous theorem, at last proved by Andrew Wiles, gives rise to an infinite number of unsolvable Diophantine equations: while $x^2 + y^2 = z^2$ has lots of solutions, for example, $x = 3$, $y = 4$ and $z = 5$; or $x = 5$, $y = 12$ and $z = 13$, and (infinitely) many more; $x^3 + y^3 = z^3$ has no such solutions at all, and the same is true when you replace the superscript 3 by any larger number. The work of Martin Davis provided the keystone from which the Russian Yuri

[8] M. Davis, *Computability and Unsolvability* (New York, 1958).

Matiyasevich eventually solved the problem, nearly twenty years later. John subsequently published a paper in the area with which he was delighted, as will be seen.

Shortly afterwards he met the distinguished English logician, John Myhill. Together they wrote another fundamental paper, this time on effective operations on partial recursive functions, which is now known as the Myhill–Shepherdson Theorem.[9]

In these years Heilbronn had Albrecht (Ali) Fröhlich as his first research student, of course in number theory. Fröhlich was a mature-age student, having worked as a carpenter and electrician in Jerusalem. Shepherdson and he wrote a short paper on the factorisation of polynomials, followed by a seminal paper,[10] which would be taken up years later when Effective Mathematics was defined, and extensively developed, initially by Nerode and Metakides from 1974.

The setting is as follows. Traditional mathematics up to the first half of the twentieth century was concerned about what is, so to say, true in the abstract, or absolutely true, though it should be added that mathematicians such as Kronecker tried to give practical methods—algorithms in modern parlance. Through the work of Brouwer and subsequently Alan Turing, attention turned more generally to what problems could be solved, as we would now say, using a computer, as opposed to just having a (possibly rather philosophical) mathematical proof that there was a solution. (One of the differences in the mathematics is that mathematicians were, and remain, very fond of the law of the excluded middle, which says of any proposition that it is either true or false, though they may not know, nor even be able to know, which is the case. Effective mathematics largely eschews this law.) Computable algebra, which is part of effective mathematics, occupies itself with knowing which problems have answers that are accessible from a (powerful enough) computer. Although several mathematicians had earlier touched on questions that involved what we now call 'effective methods', Fröhlich and Shepherdson's paper was definitely pioneering; further, the paper is a model of clarity.

Perhaps under the influence of Stephan Körner, who had joined Bristol at the same time as John and who wrote a delightful book on the philosophy of mathematics, he wrote a paper on Aristotelian logic that

[9] J. Myhill and J. C. Shepherdson, 'Effective operations on partial recursive functions', *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 1 (1955), 310–17.

[10] A. Fröhlich and J. C. Shepherdson, 'Effective procedures in field theory', *Philosophical Transactions of the Royal Society*, Series A, 248 (1956), 407–32.

was published in 1956. The paper, which is not generally noted by the mathematical community, established the soundness and completeness of formal systems of syllogistic logic, and he included it among his six selected papers when he was nominated for his Fellowship of the British Academy in 1990.[11]

John's service to Mathematical Logic in Bristol was very significant: he trained a number of students who went on to be logicians; he made Bristol a place for research in mathematical logic to which many young logicians were attracted—so much so that by far the majority of the (pure) mathematicians at Bristol were logicians, though of course they, like John, taught other branches of mathematics too. He developed a department in which people could be 'thinking all day about then impossible things' that grew into published realities. Over his forty years in the department logic flourished and he and his colleagues established an M.Sc. in Logic and the Theory of Computation, as well as producing many doctors of philosophy. Although he only had about half a dozen students who completed their PhDs with him, at least three of these became lecturers at Bristol. In addition he had a number of M.Sc. students. More importantly he partly supervised, and greatly influenced, a number of other PhD students, the most notable being C. E. M. (Mike) Yates.

John's professional interests were not confined to his own university. He fully participated in the gatherings of British logicians that had been initiated in about 1955 by Arthur Prior from Manchester (whose interests were as much in philosophy as in logic). Logic had developed at a few other centres around the country, notably Manchester and Leeds, followed by Oxford, though Leicester, Cambridge and Nottingham also featured. The group was a very mixed one and formal organisation of the British Logic Colloquium only began at a meeting in Leicester in 1965. Robin Gandy from Manchester was the first president, but John Shepherdson was the second.

In 1955 John had been promoted to Reader at Bristol and in 1957 he and Margaret were married 'about 5 July', as he noted in a genealogical table he drew up for his only grandchild. Margaret and John shared a passionate love of the outdoors. John learnt to sail but Margaret was better than he, and she would not have him crewing for her. She was a feisty woman and could be quite sharp. That did not work well on a two-person sailing dinghy. They quickly realised that this was not the path to marital

[11] J. C. Shepherdson, 'On the interpretation of Aristotelian syllogistic', *The Journal of Symbolic Logic*, 21 (1956), 137–47.

harmony and subsequently sailed mostly in single-handed vessels for much of the rest of their lives.

Unfortunately Margaret seemed prone to accidents, and even on her wedding day she turned up at the Wills Hall in the University of Bristol with her arm in a sling, having broken it while John and she were climbing in the nearby Cheddar Gorge. The wedding was a civil ceremony since John was, and remained, a staunch atheist.

Before they started a family, Margaret worked as a research assistant and postdoctoral fellow at Bristol University. In 1958–9 she accompanied John to the USA where he was a Visiting Associate Professor at the University of California, Berkeley, thanks to Alfred Tarski and Leon Henkin. He also met Julia and Raphael M. Robinson who, like Martin Davis, had been working on Diophantine equations, as well as Andrzej Mostowski from Warsaw (whose interest in inner models of set theory has been noted) and Hans Hermes from Münster. During that time he continued climbing, and on one expedition a friend with him suffered a severe and finally fatal accident in the first ascent of Mount Conness at the northern edge of Yosemite National Park. John ran 11 miles in three hours over broken stone terrain to get help, which was widely reported in the newspapers.

He read Michael Rabin and Dana Scott's work on finite automata—very simple (theoretical) computing machines—which led to their Turing Award in 1976 and this led him to work in the area. Angus Macintyre reported: '[Rabin and Scott] were justly proud of an intricate proof that two-way automata do not recognize more than one-way automata. Dana told me they were amazed and chagrined by a devastatingly clear and direct proof of this, produced quickly by John.' Recently Dana Scott confirmed his view of John's mathematical quickness: 'Of course John was always a quick study.' John's work on the basics of computing would continue.

In Bristol, John and Margaret had been living in Clifton in a small flat in West Mall. They then moved to a flat owned by John's colleague and friend Ernst Robert (Peter) Reifenberg (1939–64) on the edge of Clifton Gorge.[12] In November 1959 their first child, David, was born. In 1960 they moved just a short distance to the house where they lived the remainder of their lives (almost). It was a huge house and at first was divided into two, still large, flats. The Shepherdsons and the Ashdowns paid £4,000 for each

[12] Reifenberg was killed in a climbing accident in the Dolomites by a falling rock. Shepherdson wrote his obituary in the *Journal of the London Mathematical Society*, 40 (1965), 370–7.

of the two parts. Margaret Ashdown and Margaret Shepherdson reported intense discussions between the men: they would sit for hours dealing with the finances, often barely speaking, but incredibly intense. John did not enjoy this but he was not confrontational; he wanted things to be settled amicably. His meticulousness about household accounts, in particular those from utility companies, meant he spent much time, usually on the telephone but also writing letters, putting the companies straight—and driving them mad since he was better informed than they were—about his bills. He was relentless in his pursuit of proper accounting.

After having children Margaret returned to part-time, and then full-time, work in Bristol as a lecturer in biochemistry and genetics at the University, and subsequently what later became the University of the West of England. She officially retired in the 1990s, but remained active professionally on a part-time basis. Earlier, when the children were growing up, John frequently went rock climbing and the oldest child, David, soon learnt to climb. Their daughter Jane reported:

> They didn't seem like anyone else's parents, they were young at heart and defied convention … life wasn't about indulging young children in what they wanted, it was about showing them all that nature had to offer … childhood holidays were to the Outer Hebrides, climbing Shehalion, hiking in Death Valley (California), and sailing on Loch Lomond. … I'm ashamed to say that I sometimes longed for the traditional English beach holidays of my schoolmates.

When the parents went yachting the children were left to play on their own, perhaps messing about in (small) boats. Academic pursuits, however, were very different for the children. None of them inherited John's mathematical abilities and aptitudes. This had two effects: the practical one that he was hopeless at trying to teach them their school mathematics; he could not come down to their level and Margaret took over that task; but it also meant that the world of higher mathematics and his professional life was a closed book to them. The memorial service in February 2015 was therefore a revelation to the family as well as revealing something of John's family life to his logic colleagues.

John's professional life flourished and he worked on and off on Gödel's famous incompleteness theorems, of which the most well known is characterised as saying there is a sentence of formal arithmetic that is true but not provable. This led Ray Smullyan, in his beautiful first book, to give an 'extremely ingenious' proof due to John.[13]

[13] R. Smullyan, *Theory of Formal Systems* (revised edition) (Princeton, NJ, 1961), p. 140.

In 1963, John wrote a paper with H. E. Sturgis that dramatically simplified the theoretical description of computable functions.[14] When Alan Turing had formulated his notion of what is now called a Turing machine and which paved the way for modern computers, he envisaged it as having a paper tape on to which symbols were written, much like ticker-tape machines, which have patterns of holes punched into them to represent letters and numbers. The Turing machine operates by shifting between various possible states depending on what it 'reads' on the tape. (Those who were using computers in the 1960s and earlier will be familiar with actual machines that had such tapes.) Turing was adept at reading such physical tapes, but the modern computing machine works differently. Although nowadays it is often hidden behind layers of sophisticated diagrams on the screen, the basic machine has a number of 'registers' that hold numbers.[15] This is the form of the Shepherdson–Sturgis machine or, as it is often called, an Unlimited Register Machine. It has a program that operates by reading an instruction, then looking at a particular register, adding or subtracting one from the number in that register, and then going on to the next instruction. There is just one other kind of instruction, which is more sophisticated: depending on the number it finds in the register it can jump to a different part of the program. That really is all there is to it. Nevertheless, such machines are just as powerful as Turing machines. They are simply much easier to understand—and teach.

The paper itself is beautifully simple to read but the meticulousness one associates with John shines through. Scrupulous attention is also given to the contributions made by other authors to the complex sequence of predecessors of the Shepherdson–Sturgis machines. The following year John was promoted to a professorship at Bristol. Later he would be elevated to the Wills Professorship of Mathematics from 1977.

The mid-1960s saw John take up yet another branch of mathematical logic. His interest in models of set theory has already been mentioned; now he turned to arithmetic, or rather to so-called non-standard models of arithmetic. It turns out that besides the ordinary numbers, 0, 1, 2, 3 and so on, fulfilling the usual laws of arithmetic, especially mathematical induction, there are other structures that satisfy the same laws although

[14] J. C. Shepherdson and H. E. Sturgis, 'Computability of recursive functions', *Journal of the Association for Computing Machinery*, 10 (1963), 217–55.

[15] There are two important aspects in which the theoretical machines differ from practical ones: the registers can contain arbitrarily large numbers and there can be as many registers as you like (though fixed for a given machine), unlike a 'real' machine, which is limited in size in both respects.

they can look very different. Such models give us insight into how 'ordinary' arithmetic behaves. John produced models for so-called fragments of number theory, that is, systems with only some of the usual laws, which attracted considerable attention.[16] This work seems to have been the precursor of a lot of activity in the area.

About that time I was a fellow at St Catherine's College, Oxford, and invited John and Margaret to dinner. They arrived, after dark, having travelled from Bristol. They were running late. John encouraged Margaret to take the dark path, which would be a short cut to the dining hall. They quickly discovered that the 'dark path' was actually a long ornamental pool running in front of one of the buildings. Fortunately they had been sailing that afternoon and so had their sailing clothes in the car. They changed but, as they came in, late, to the high table there was a trail of water left behind. The Master, Alan Bullock, who was presiding, was a very convivial soul who greeted them warmly and, despite any discomfort, they did enjoy the evening: it was certainly a memorable one.

John's professional logic interests continued to develop throughout the 1960s, and the advent of Research Training and Support Grants from 1964 to 1972 meant that, besides having a good crew of logicians in Bristol, he was able to invite distinguished visitors from home and overseas to spend time there. His publications during this period reflect the great diversity of his interests—and competencies. At least one of the visitors would have liked to have stayed in Bristol but the department was already replete with logicians; John Shepherdson gathered together the largest such group that has ever been employed in one place in Britain.

The list of visitors was remarkable for its quality and the variety of their countries. It included Bill Boone, Haskell B. Curry, John N. Crossley, Martin Davis, Erwin Engeler, Leon Henkin, Laslo Kalmar, Georg Kreisel, Paul Lorenzen, David Luckham, Ken MacAloon, Joan and Yiannis Moschovakis, Andrzej Mostowski, John's old friend John Myhill, Hartley Rogers, Jr., Gerry Sacks, Dana Scott, Ernst P. Specker and Alfred Tarski. When Kreisel was visiting Bristol, I regularly took a carload of research students across from Oxford to listen. Kreisel's lectures were incomprehensible but hypnotic. I later wrote to John (after Margaret's death) reminiscing about Kreisel: 'What a challenge! But I think we both (you and I) came out of that quite well despite all he could do.'

[16] See J. C. Shepherdson, 'Non-standard models for fragments of number theory', in J. W. Addison, L. Henkin and A. Tarski (eds.), *Theory of Models (Proceedings of the 1963 International Symposium, Berkeley)* (Amsterdam, 1965), pp. 342–58.

John's interest in computability went further than most people's as far as fundamentals are concerned. The primary interest was always in computations on numbers but John also considered arbitrary structures. During this period John also revisited Gödel's Second Incompleteness Theorem. This move was influenced by Kreisel who insisted that discussion of the 'unprovability of consistency' was very sloppy indeed.

1966 saw John again in Berkeley and then in 1971 he went to Australia, where I had moved in 1969, for the first of three visits. When he first arrived in Australia he had immediately gone to the Great Barrier Reef, where he went scuba diving before travelling on. Together with four other logicians I had planned a trip to central Australia. The arrangement was that John would be picked up at the airport and then the party would proceed to Ayer's Rock (now known as Uluru) in the very centre of Australia. It worked like clockwork and John was a wonderful addition to the party. I described him then as 'modest, lean, and by far the most senior of us and also an accomplished rock climber'. John stayed till October as a Visiting Professor at Monash University in Melbourne, where he remarked that the students seemed to be about six months ahead of corresponding students in the UK.

John had never even started a PhD but in 1981 he took out an Sc.D. from Cambridge, presumably to put himself into line with most other academics who did have a doctorate, especially those in the USA. In 1982 John published a paper with Verena Huber-Dyson and J. P. Jones following Matiyasevich's proof of the general unsolvability of Diophantine equations (mentioned above), and this paper amused him very much.[17] The paper gives explicitly an undecidable formula that involves eleven alternations of quantifiers,[18] one of the most complicated sequences of quantifiers explicitly written out. The formula is equivalent to having a solution to a (very large) set of Diophantine equations.

He spent three summers (1973, 1975 and 1979) at the IBM Research Laboratories at Yorktown Heights where interest in the new area of Logic Programming was growing, and John's interests turned there too. Computing had begun with Turing's abstract notions that were inspired by, and closely related to, mathematical logic. It was therefore not surprising when computer scientists started to use logic as a programming language; the problem had been how to do that. There was a difficulty,

[17] V. H. Dyson, J. P. Jones and J. C. Shepherdson, 'Some Diophantine forms of Gödel's theorem', *Archiv für Mathematische Logik und Grundlagenforschung*, 22 (1982), 51–60.

[18] Quantifiers are qualifiers of the form 'for all *x*' or 'there exists *x*'.

however, in that although the work of Brouwer and others had led to the construction of formal languages that avoided the uncertainties mentioned above regarding the law of the excluded middle, pinning the details down enough to get a workable programming language did not always give a nice theoretical language. Nevertheless the computer scientists ploughed on, and a programming language called PROLOG (programming in logic) was created.

There were flaws in the theory and John, in his usual thorough way, teased them out. One in particular involved a so-called completeness theorem. The proof in John Lloyd's book had a gap,[19] but it was John Shepherdson who filled it. John became quite famous on the topic of 'Mistakes in logic programming'.[20] However, again in John's non-confrontational way, he went on to write a number of papers with John Lloyd. The latter's only regret was that Shepherdson did not recruit him to Bristol. In 1984 John ran a 'Discussion Meeting' at the Royal Society with Michael Atiyah and Tony Hoare on Mathematical Logic and Programming Languages; such was his renown in the area. Later that year the British Logic Colloquium was held at Bristol, which he chaired, having been an active, indeed founding member of the group.

The following year his genetic heritage caught up with him. Angina had always been a problem and in 1985 he was told he needed heart bypasses. There are two stories around his admission to hospital. One is that he rode his bicycle there, the other that he had to be restrained from doing so. In the event he had a sextuple bypass and lived another thirty years; his surgeon had told him he would live twenty. Nevertheless John thereafter always said he was enjoying every day of the life extension he had been granted. He was always very slim and, after this heart attack, he adopted a fat-free diet that was strictly enforced by Margaret. It was a wise move and he retained a sparse figure throughout the rest of his life.

In 1990 John was elected a Fellow of the British Academy having been nominated by a galactic and mixed group of philosophers and mathematicians: Tim J. Smiley, Margaret A. Boden, Stephan Körner, John R. Lucas, D. H. Mellor and Peter F. Strawson: 'He is especially noteworthy for the catholicity of his interests and so for his original

[19] J. W. Lloyd, *Foundations of Logic Programming* (Berlin and New York, 1984; second edition, 1987).

[20] An early version of his paper, J. C. Shepherdson, 'The role of standardising apart in logic programming', *Theoretical Computer Science*, 129 (1994), 143–66, when circulated in manuscript had an earlier part to the title: 'Mistakes in Logic Programming'.

contributions to topics that are of interest to philosophers as well as mathematical specialists.'

John's interest in Logic Programming continued, and he published papers in the area right through until 2001. He also got interested in other branches of mathematical logic and computing. One new interest was fuzzy logic, and he made valiant attempts to make a rigorous discipline out of what had emerged as a practical engineering idea.

In 1991 he visited Melbourne once more. There we worked together on proof theory. Shortly thereafter (in 1992) John remarked at the British Logic Colloquium in his honour that I had shown him that 'proof-theory isn't as forbidding as I always imagined it to be'. When John stayed with us we worked, sometimes at logic, sometimes putting up replacement filigree work as often adorns Victorian-era houses in Melbourne. He worked hard at everything and was a relentless taskmaster. I had first-hand experience of his meticulousness and his obsession with accuracy. No waving of hands (as mathematicians label loose reasoning) was permitted; everything had to be correct, and clearly so. The work, whose aim was to extract computer programs from logical proofs (and which was much more reliable than logic programming), involved taking a very esoteric system invented by Jean-Yves Girard and working out its mechanism so that it could be used on the familiar systems of predicate or first-order logic. The resulting paper was elegant, readable and surprisingly useful, and would not have been half so good without John's input.[21]

In addition to working together on mathematical logic we made expeditions into the outback where, one day, we walked over 15 miles in Wyperfeld National Park. He literally took that in his stride. However, navigation on land was certainly not John's strongest suit. He said in his retirement speech in 1992: 'I have been lost in the mountains often enough [in Britain].' On one visit to Australia, when we were living in the bush, he went out on his own one afternoon for a walk in the surrounding homogeneous, and almost trackless, dry sclerophyll forest. It was getting dark when the phone rang. John sheepishly admitted that he had found a track and a house—he was a couple of miles off course, though he could have walked barely 6 miles.

As well as visiting Monash, in 1991 he was also a Visiting Professor and Visiting Scientist at Princeton, the University of California at Berkeley

[21] J. N. Crossley and J. C. Shepherdson, 'Extracting programs from proofs by an extension of the Curry-Howard process', in J. N. Crossley, J. B. Remmel, R. I. Shore and M. E. Sweedler (eds.), *Logical Methods* (Ithaca, NY, 1992; Boston, MA, 1993), pp. 222–88.

and IBM Yorktown Heights (once more). It was perhaps about this time, on the verge of his retirement, that he enunciated his long-standing question of whether he should have stayed at Bristol throughout his career. Although he said on retirement that 'it probably wasn't a good idea to stay in one place so long; I wouldn't recommend it to you', he admitted that he had found Bristol very congenial and the British way of life more suited to him than the American. His nagging doubt was that he might have been stimulated to even greater achievements if he had lived in the American hothouses of logic such as Berkeley and Stanford, which also had 'the lure of the Sierras and the ocean'. Perhaps like many other high achievers that was not how he saw himself, but rather as an underperformer. By contrast, what was very striking was his recounting that he had 'had an extremely enjoyable time in this department [at Bristol]', but then John always seemed to manage to enjoy life to the full. Even though he worked hard he always seemed to me very relaxed about spending time on leisure activities and found ample time for his sporting interests, most notably skiing and climbing. His skiing continued, almost every year in the company of one of his oldest friends, John Marstrand, an analyst from the Bristol department described as a 'very interesting character'. John Shepherdson was delighted when he got a free ski pass at age 75 or 80. He continued skiing until he was 84 (in 2010). His climbing also continued, in particular with his former student, later colleague, Mike Gladstone and others who formed what Margaret called the '"Last of the Summer Wine" crowd',[22] though they named themselves the Moss Ghyll group after an epic outing there.[23]

John liked classical music and opera, but in general he did not make the effort to go to see operas. There was at least one exception. Margaret and he went to Glyndebourne; he complained that he had to hire a dinner suit and that Margaret bought a new gown. His other indoor pursuits were dominated by chess throughout his life, though his children played him at table tennis for which John would just pick up a log ready for the fire to use, with considerable success, as a bat. His daughter Jane's husband, Barry, played chess with him regularly, and beat him only twice; once in the last few months of John's life. He was not a party animal by any means, though he did hold parties for the department and when the British Logic Colloquium was held in Bristol, but his conversation was always 'bright

[22] *The Last of the Summer Wine* was a long-running television comedy series about old men and their travails in Yorkshire.
[23] Moss Ghyll is in the Lake District, near Scafell.

and entertaining, with a still-waters-running-deep reservoir of dry wit which was of great amusement when it bubbled up'. He was the perfect houseguest, appearing and disappearing on cue and contributing in many ways, not least by quietly not interfering.

When he was close to retirement he took up cooking as Margaret was still working. He told stories against himself, for example, cooking salmon at 200°, which was unsuccessful since the temperature should have been Celsius, not Fahrenheit. His daughter Jane relates that he also found following recipes difficult because of their imprecision. 'How much is a glass of sherry?' he would ask her, for he had to know exactly how many millilitres were meant.

When Margaret became ill and developed dementia he devoted his life to looking after her during her last years; he fed and nursed her as long as he possibly could. He would not even consider sending her to a nursing home until it was evident that he just could not cope any more. Eventually he acceded to her being transferred to a home and she died only a few months afterwards, in the middle of 2014. John was distraught. The Shepherdsons had already bought the other flat from the Ashdowns and now he was living on his own in the huge house. He felt it was time to move and he did so, but only to the newish block of flats next door. He had hardly got there when he was diagnosed with an inoperable sarcoma. He died peacefully shortly afterwards on 8 January 2015, just a few months after Margaret.

He had always been a firm, even 'devout', atheist and he was determined his body should not go to waste. He left it to the Bristol University Medical School and, although he had wanted his brain to go to Parkinson's,[24] unfortunately the drugs that he was on at the end of his life meant that they could not take it. A memorial service was held on 21 February 2015 and was conducted by a member of the British Humanist Society. It was attended by a Who's Who of British logic as well as local friends and family. The service surpassed many a religious one in terms of its simplicity, sincerity, devotion, visible affection and the love displayed.

John Shep, for thus he was always known, was masterly in a non-confrontational way. He had great mastery of an unusually wide range of mathematical logic—and beyond. He was meticulous to a fault. His modesty was palpable, but that meant that most of us did not fully appreciate the wealth and breadth of his life and work. A favoured few

[24] The British charity dedicated to the malady. John did not suffer from the disease.

also enjoyed his outdoor adventures, not least Margaret. As his daughter Jane has said: 'They had a wonderful life.'[25]

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Note. I am extremely grateful to family, friends and colleagues of John Shepherdson for providing materials and information John revealed only selectively and sparsely, if at all. As Angus Macintyre said: 'John's versatility makes it almost impossible for just one person to cover it all.' So special thanks to the Shepherdson children, David, Jane (especially) and Judith, and numerous logicians, in particular Angus Macintyre, Philip Welch and Wilfrid Hodges.

[25] No complete list of John Shepherdson's publications is available. He wrote over sixty papers as well as numerous reviews: most of the papers are listed on the *Mathematical Reviews*® website <http://www.ams.org/mr-database> (accessed 29 January 2016).